AP Calculus AB

Derivatives of Trig Functions

1) $f(x) = \tan x$

$f(x) = \frac{\sin x}{\cos x}$

$f'(x) = \frac{\cos x(\cos x) - \sin x(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$\underline{f'(x) = \sec^2 x}$

2) $f(x) = \cot x$

$f(x) = \frac{1}{\tan x} = (\tan x)^{-1}$

$f'(x) = -(\tan x)^{-2} \cdot \sec^2 x$

$f'(x) = -\frac{1}{(\tan x)^2} \cdot \sec^2 x$

$f'(x) = -\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} = -\frac{1}{\sin^2 x}$

$\underline{f'(x) = -\csc^2 x}$

3) $f(x) = \sec x = \frac{1}{\cos x}$

$f(x) = (\cos x)^{-1}$

$f'(x) = -(\cos x)^{-2} \cdot (-\sin x)$

$f'(x) = \frac{1}{\cos^2 x} \cdot \sin x)$

$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$\underline{f'(x) = \sec x \tan x}$

4) $f(x) = \csc x = \frac{1}{\sin x} = (\sin x)^{-1}$

$f'(x) = -(\sin x)^{-2} \cdot \cos x$

$f'(x) = -\frac{1}{\sin^2 x} \cdot \cos x$

$= -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$

$\underline{f'(x) = -\csc x \cot x}$

5) $f(x) = 11x + 3\cos x$

$f'(x) = 11 - 3\sin x$

6) $y = \frac{\boxed{5\cos x}}{1 - \sin x}$

$\frac{dy}{dx} = \frac{(1-\sin x)(-5\sin x) - (5\cos x)(-\cos x)}{(1-\sin x)^2}$

$= \frac{-5\sin x + 5\sin^2 x + 5\cos^2 x}{(1-\sin x)^2}$

$= \frac{5(-\sin x + \sin^2 x + \cos^2 x)}{(1-\sin x)^2}$

$= \frac{5(-\sin x + 1)}{(1-\sin x)^2} = \boxed{\frac{5}{1-\sin x}}$

7) $f(x) = 3x^2 + 9x - 3$

Point: $(-1, -9)$

slope: $f'(x) = 6x + 9$

$f'(-1) = 3$

$L(x) = -9 + 3(x+1)$

$L(-0.9) = -9 + 3(0.1)$

$= -9 + 0.3$

$= -8.7$

$\boxed{f(-0.9) \approx -8.7}$

8) $\sqrt{147} \sim f(x)=\sqrt{x}$

point: $(144, 12)$

slope: $f'(x) = \frac{1}{2\sqrt{x}}$

$f'(144) = \frac{1}{24}$

$L(x) = 12 + \frac{1}{24}(x-144)$

$L(147) = 12 + \frac{1}{24}(3)$

$\boxed{f(147) \approx 12\frac{1}{8}}$

9) $L(x) = 5 + 6(x-3)$

$L(2.9) = 5 + 6(-0.1)$

$= 5 - 0.6$

$\boxed{f(2.9) \approx 4.4}$

10) $y = \boxed{3x}\,\sin x$

$y' = \boxed{3x}\cos x + 3\sin x$

$y'' = 3x(-\sin x) + 3\cos x \quad + 3\cos x$

$y'' = -3x\sin x + 6\cos x$

11) $f(x) = \boxed{6\sin x}\,\cos x$

$f'(x) = 6\sin x(-\sin x) + 6\cos x \cos x$

$f'(x) = -6\sin^2 x + 6\cos^2 x$

12) $f(x) = 4\sec x + 5\cot x$

$f'(x) = 4\sec x\tan x - 5\csc^2 x$

13) $y = 8\cot x$

$y' = -8\csc^2 x = -8(\csc x)^2$

$y'' = -16(\csc x)\cdot(-\csc x\cot x)$

$y'' = \underline{16\csc^2 x\cot x}$

14) $f(x) = \boxed{\csc x}\,\sec x$

$f'(x) = (\csc x)[\sec x\tan x] + (\sec x)[-\csc x\cot x]$

$= \frac{1}{\cancel{\sin x}}\left(\frac{1}{\cos x}\cdot\frac{\cancel{\sin x}}{\cos x}\right) + \frac{1}{\cancel{\cos x}}\left[-\frac{1}{\sin x}\cdot\frac{\cancel{\cos x}}{\sin x}\right]$

$\boxed{f'(x) = \sec^2 x - \csc^2 x}$

15) $f(x) = \tan 3x - \cot 3x$

$f'(x) = \sec^2 3x\cdot(3) + \csc^2 3x\,(3)$

$\boxed{f'(x) = 3\sec^2 3x + 3\csc^2 3x}$

16) $f(x) = \dfrac{\boxed{\tan x}}{\cos x - 4}$

$$f'(x) = \frac{(\cos x - 4)\cdot(\sec^2 x) - (-\sin x)(\tan x)}{(\cos x - 4)^2}$$

$$f'(x) = \frac{\sec x - 4\sec^2 x + \sin x \tan x}{(\cos x - 4)^2}$$

17) $f(x) = \boxed{\sec x}\,\tan x$

$$f'(x) = \sec x\,(\sec^2 x) + \tan x\,(\sec x \tan x)$$

$$\boxed{f'(x) = \sec^3 x + \sec x \tan^2 x}$$